# Aerodynamic Parameter Identification for an Airborne Wind Energy Pumping System \*

G. Licitra<sup>\*,\*\*\*\*</sup> P. Williams<sup>\*</sup> J. Gillis<sup>\*\*\*</sup> S. Ghandchi<sup>\*</sup> S. Sieberling<sup>\*</sup> R. Ruiterkamp<sup>\*</sup> M. Diehl<sup>\*\*\*\*</sup>

\* Ampyx Power B.V., The Hague, Netherlands g.licitra@ampyxpower.com, p.williams@ampyxpower.com, s.ghandchi@ampyxpower.com, s.sieberling@ampyxpower.com r.ruiterkamp@ampyxpower.com \*\*\* Katholieke Universiteit Leuven, Leuven, Belgium joris.gillis@kuleuven.be \*\*\*\* Dept. of Microsystems Engineering & Dept. of Mathematics, University of Freiburg, Freiburg Im Breisgau, Germany moritz.diehl@imtek.uni-freiburg.de

**Abstract:** Airborne Wind Energy refers to systems capable of harvesting energy from the wind by flying crosswind patterns with a tethered aircraft. Tuning and validation of flight controllers for AWE systems depends on the availability of reasonable a priori models. In this paper, aerodynamic coefficients are estimated from data gathered from flight test campaign using an efficient multiple experiments model based parameter estimation algorithm. Data fitting is performed using mathematical models based on full six degree of freedom aircraft equations of motion. Several theoretical and practical aspects as well as limitations are highlighted. Finally, both model selection and estimation results are assessed by means of R-squared value and confidence ellipsoids.

Keywords: Airborne Wind Energy, Model-Based Parameter Estimation

# 1. INTRODUCTION

Airborne wind energy (AWE) is a novel technology emerging in the field of renewable energy systems. The idea of using tethered aircraft for wind power generation, initially motivated by Loyd (Loyd, 1980), has never been closer to a large scale realization than today. High power-to-mass ratio, capacity factors, flexibility and low installation costs with respect to the current established renewable technologies, encourage both academia and industries to invest on these systems. However, complexities arise significantly in terms of control, modeling, identification, estimation and optimization. Among the different concepts in the landscape of AWE (Diehl, 2013), one interesting case study is the so called *pumping mode* AWE system (AWES). In a *pumping mode* AWES, the airplane delivers a high tension on the tether which is anchored to a ground-based generator. During *production phase*, the tether tension is used to rotate a drum that drives an electric generator. Due to finite tether length, a *retraction phase* is needed, hence the tether is wound back by changing the flight pattern in such a way that less lifting force is produced, with significant lower energy investment than what was gained during the production phase. A pumping mode AWES is being developed by Ampyx Power (AP, 2016).



Fig. 1. Example of a pumping cycle with a *production* and *retraction phase* 

The airborne component is referred to as a *PowerPlane*. An artist's rendering of the two main phases of a *pumping* mode AWES is shown in Fig. 1. The *PowerPlane*, is a high lift aircraft designed for extremely challenging operational environment including high tension from the tether and high accelerations that arise during the pattern. A concept design of the *PowerPlane*  $3^{rd}$  generation (AP3) is shown in Fig. 2. System simulators require adequate models of the entire system, including the *PowerPlane*. Existing analysis tools such as Computational Fluid Dynamics (CFD) (Versteeg and Malalasekera, 2007) or lifting line (Anderson Jr, 2010) are able to provide initial estimates of parameters, but in most cases the full dynamic effects on the real system have to be determined through flight testing. In this case, the main issue is to describe mathematically the aerodynamic forces and moments as a function of

<sup>\*</sup> This research was supported by Support by the EU via ERC-HIGHWIND (259 166), ITN-TEMPO (607 957), ITN-AWESCO (642 682) and by DFG in context of the Research Unit FOR 2401.



Fig. 2. CFD analysis of  $3^{rd}$  generation *PowerPlane* 

airspeed, angle of attack, angle of side slip and body rotation rates. Usually, Taylor series expansion are used to represent the aerodynamic properties. The parameters of the expansion are known as *aerodynamic derivatives* (or simply *derivatives*) and for conventional aircraft they are mainly used for control system design and handling qualities studies. For AWES, accurate modeling also enables computation of reliable trajectories by means of optimal control problems (OCPs) (Horn et al., 2013; Licitra et al., 2016), as well as design of advanced feedback controls such as non linear model predictive control (NMPC) (Zanon et al., 2013). In the aerospace field, it is the current practice to retrieve *derivatives* by empirical data obtained from similar aircraft configurations or with tools based on CFD, augmenting and verifying them by wind tunnel tests. For standard aircraft configurations such methods for obtaining aerodynamic characteristics is generally in good agreement with experimentally obtained values. However, CFD and wind tunnel tests are expensive and time consuming, and tend to be limited to static effects. Therefore, an intensive flight test campaign must be set in order to gain additional insight about aerodynamic properties. In this paper, aerodynamic derivatives are determined by means of time domain system identification techniques using measurements coming from real flight tests.

The paper is organized as follows. In Section II, model structure is retrieved from a high fidelity aircraft model augmented with description of model assumptions as well as neglected dynamics. Section III presents an efficient formulation of multiple experiment model based parameter estimation (MBPE) algorithm. In Section IV, data fitting is computed first with simulated experiments where the block structure of the nonlinear program (NLP) is shown, observation with respect to aircraft inertia are provided and confidence ellipsoids are introduced. Finally, data fitting is computed with the real experiments where the reliability of both model and estimates are assessed respectively by the *R-squared* value and confidence ellipsoids.

#### 2. POWERPLANE MATHEMATICAL MODEL

# 2.1 Model Selection

A *pumping mode* AWES can be modeled via Differential Algebraic Equations (DAEs) described both by minimal (Williams et al., 2007, 2008) and non-minimal coordinates (Gros and Diehl, 2013). By means of Lagrangian mechanics one can build the equations of motion for a six degree

of freedom (DOF) tethered aircraft model. For parameter estimation purposes, let us consider the translational and rotational dynamics of a *pumping mode* AWES expressed in the body-fixed reference frame:

$$m \cdot \dot{\boldsymbol{v}}_{\boldsymbol{b}} = \boldsymbol{F}_{\boldsymbol{c}} + \boldsymbol{F}_{\boldsymbol{p}} + \boldsymbol{F}_{\boldsymbol{a}} + \boldsymbol{F}_{\boldsymbol{g}} - m\left(\boldsymbol{\omega}_{\boldsymbol{b}} \times \boldsymbol{v}_{\boldsymbol{b}}\right)$$
(1a)

$$\mathbf{J} \cdot \boldsymbol{\omega}_{\boldsymbol{b}} = \boldsymbol{M}_{\boldsymbol{c}} + \boldsymbol{M}_{\boldsymbol{p}} + \boldsymbol{M}_{\boldsymbol{a}} - (\boldsymbol{\omega}_{\boldsymbol{b}} \times \mathbf{J} \cdot \boldsymbol{\omega}_{\boldsymbol{b}})$$
(1b)

where  $\boldsymbol{v_b} = [u, v, w]^T$  and  $\boldsymbol{\omega_b} = [p, q, r]^T$  are respectively the translational and rotational speed vector, m the aircraft mass and  $\mathbf{J}$  the inertia dyadic of the aircraft. The aircraft is subject to forces  $F_{(.)}$  and moments  $M_{(.)}$  coming from the cable, propellers, gravity and the interaction between aircraft with the air mass is denoted by  $F_a = [X, Y, Z]^T$  and  $M_a = [L, M, N]^T$ . Notice that, although pumping mode AWES does not assume any propellers during power generation phase, they are present in the studied *PowerPlane* design for assisting launch and landing as well as performing general purpose untethered flights.

In order to identify the aerodynamic forces  $F_a$  and moments  $M_a$ , one has to discard or have good models of the other contributions. Hence, the flight test campaign aimed to identification of aerodynamic models should be performed without cable such that the cable does not interfere with the overall aircraft dynamics. Additionally, propellers are switched off whenever an excitation signal occurs in order to decouple the uncertainty in thrust effects on the aerodynamic parameter estimation, simplifying (1) to

$$m \cdot \dot{\boldsymbol{v}}_{\boldsymbol{b}} = \boldsymbol{F}_{\boldsymbol{a}} + \boldsymbol{F}_{\boldsymbol{g}} - m\left(\boldsymbol{\omega}_{\boldsymbol{b}} \times \boldsymbol{v}_{\boldsymbol{b}}\right)$$
(2a)

$$\mathbf{J} \cdot \dot{\boldsymbol{\omega}}_{\boldsymbol{b}} = \boldsymbol{M}_{\boldsymbol{a}} - (\boldsymbol{\omega}_{\boldsymbol{b}} \times \mathbf{J} \cdot \boldsymbol{\omega}_{\boldsymbol{b}}) \tag{2b}$$

In general, the aerodynamic forces and moments are all dependent on the time history of the aircraft state in time, which mean that if the pitch moment M depends on the pitch rate q only, then:

$$M(t) = f(q(t)), t \in (-\infty, \tau]$$
(3)

In theory, the function in time q(t) can be replaced by the following Taylor series:

$$q(t) = q(\tau) + \sum_{i=1}^{\infty} \frac{1}{i!} \frac{\partial^i q}{\partial \tau^i} (t - \tau)^i$$
(4)

i.e. that the whole information regarding the parameter history q is captured, if we were able to compute all the possible derivatives. However, for subsonic flight the influence of the derivatives is bounded and can be neglected with some exception (Mulder et al., 2000). Furthermore, the aerodynamic properties can be normalized with respect to the dynamic pressure  $\bar{q} = \frac{1}{2}\rho V^2$  with  $\rho$  the free-stream mass density, V the free-stream airspeed, and a characteristic area for the body

$$\boldsymbol{F_a} = \bar{q} \, S \cdot [C_X \,, \, C_Y \,, \, C_Z]^T \tag{5a}$$

$$\boldsymbol{M_a} = \bar{q} \, \boldsymbol{S} \cdot [\boldsymbol{b} \, \boldsymbol{C_l} \,, \, \bar{\boldsymbol{c}} \, \boldsymbol{C_m} \,, \, \boldsymbol{b} \, \boldsymbol{C_n}]^T \tag{5b}$$

In (5)  $S, b, \bar{c}$  are respectively reference wing area, wing span and mean aerodynamic chord while  $C_X, C_Y, C_Z$ denote the forces and  $C_l, C_m, C_n$  the moment coefficients. During the system identification flight test, excitation signals are performed only along one axis in open-loop, keeping trimmed the other dynamics. Therefore, one can decouple the full dynamics in two sets of independent dynamics, three equations for the translational motion and three for the rotational one. Still, from an optimization point of view, embedding the full mathematical model shown in (2), would add complexity to the estimation algorithm without any particular benefit since the trimmed dynamics will not provide meaningful experimental data. As far as it regards the translational dynamics (2a), denoting  $\theta$  and  $\phi$  respectively pitch and roll angle and g gravity, the decoupled equations of (2a) will be

$$\dot{u} = \frac{X}{m} - qw + rv - g\sin\theta \tag{6}$$

$$\dot{v} = \frac{Y}{\underline{m}} - ru + pw + g\cos\theta\sin\phi \tag{7}$$

$$\dot{w} = \frac{Z}{m} - pv + qu + g\cos\theta\cos\phi \tag{8}$$

while roll, pitch and yaw dynamics are retrieved from (2b)

• Roll dynamics

$$\dot{p} = -\frac{J_z L + J_{xz} N - q r J_{p_1} + p q J_{p_2}}{J_{xz}^2 - J_x J_z}$$

$$J_{p_1} = (J_{xz}^2 + J_z^2 - J_y J_z)$$

$$J_{p_2} = J_{xz} \cdot (J_x - J_y + J_z)$$
(9)

• Pitch dynamics

$$\dot{q} = \frac{M + J_{xz} \left(r^2 - p^2\right) + p r \left(J_z - J_x\right)}{J_y} \tag{10}$$

• Yaw dynamics

$$\dot{r} = -\frac{J_{xz}L + J_xN + p q J_{r_1} + q r J_{xz} J_{r_2}}{J_{xz}^2 - J_x J_z}$$

$$J_{r_1} = (J_x^2 + J_{xz}^2 - J_x J_y)$$

$$J_{r_2} = (J_y - J_x - J_z)$$
(11)

where  $J_x, J_y, J_z$  are the moments of inertia with respect to the axis specified by the subscript while  $J_{xz}$  is the product of inertia.  $J_{yz}$  as well as  $J_{zy}$  are zero due to the symmetry of the aircraft.

In this paper we focus on the  $2^{nd}$  generation PowerPlane (AP2) shown in Fig. 3. For this aircraft, coefficients defined in (5) are broken down into a sum of terms which depend on normalized body rates  $\hat{p}, \hat{q}, \hat{r}$  , angle of attack  $\alpha$  and of side slip  $\beta$ , as well as aileron  $\delta_a$ , elevator  $\delta_e$  and rudder  $\delta_r$ deflections:

$$C_X = C_{X_p}\hat{p} + C_{X_q}\hat{q} + C_{X_r}\hat{r} \tag{12a}$$

$$C_Y = C_{Y_\beta}\beta + C_{Y_p}\hat{p} + C_{Y_q}\hat{q} + C_{Y_r}\hat{r}$$
(12b)

$$C_Z = C_{Z_\beta}\beta + C_{Z_p}\hat{p} + C_{Z_q}\hat{q} + C_{Z_r}\hat{r}$$
(12c)

$$C_{Y} = C_{Y_{\beta}}\beta + C_{Y_{p}}\hat{p} + C_{Y_{q}}\hat{q} + C_{Y_{r}}\hat{r}$$
(12b)  

$$C_{Z} = C_{Z_{\beta}}\beta + C_{Z_{p}}\hat{p} + C_{Z_{q}}\hat{q} + C_{Z_{r}}\hat{r}$$
(12c)  

$$C_{l} = C_{l_{\beta}}\beta + C_{l_{\delta_{a}}}\delta_{a} + C_{l_{\delta_{r}}}\delta_{r} + C_{l_{p}}\hat{p} + C_{l_{r}}\hat{r}$$
(12d)  

$$C_{L} = C_{L_{\beta}}\beta + C_{L_{\delta_{a}}}\delta_{a} + C_{L_{\delta_{r}}}\delta_{r} + C_{L_{p}}\hat{p} + C_{l_{r}}\hat{r}$$
(12d)

$$C_m = C_{m_\alpha} \alpha + C_{m_{\delta_e}} \delta_e + C_{m_q} \hat{q} + C_{m_0} \tag{12e}$$

$$C_n = C_{n_\beta}\beta + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r + C_{n_p}\hat{p} + C_{n_r}\hat{r} \quad (12f)$$

$$\hat{p} = \frac{b\,p}{2V} \,, \, \hat{q} = \frac{\bar{c}\,q}{2V} \,, \, \hat{r} = \frac{b\,r}{2V} \tag{13}$$

$$\alpha = \arctan\left(\frac{w}{u}\right), \ \beta = \arcsin\left(\frac{v}{V}\right)$$
(14)

The coefficients  $C_{i_j}$  with  $i = \{X, Y, Z, l, m, n\}$  and j = $\{\alpha, \beta, p, q, r, \delta_a, \delta_e, \delta_r, 0\}$  are the *aerodynamic derivatives* that need to be identified.

# 2.2 Model Assumption and Neglected Dynamics

Focusing exclusively on the aircraft dynamics, several assumptions are made to simplify the identification problem, and these are summarized below:



Fig. 3.  $2^{nd}$  generation PowerPlane

- By neglecting the influence of the derivatives in time shown in (4), one neglects the influence of parameter variation through time. Such influence arises from non-stationary wing-fuselage and tail interference, increasing during aggressive maneuvers (Mulder et al., 2000), in our case mainly during the *power-generation* phase. However some dynamics can be captured introducing a first-order differential equation involving angle of attack rate  $\dot{\alpha}$  (Goman and Khrabrov, 1994).
- Aircrafts have flexible modes that are neglected in (1-2) since we rely on rigid-body equations. The PowerPlane utilizes a high-strength wing with relatively high stiffness. Flexible modes need to be considered for the control system design because of possible structural-coupling issues. However, the effect of the flexible modes on aerodynamics are neglected.
- The model assumed in (12) is implicitly a function of  $\alpha$  though, estimations performed via flight tests are typically valid only for small neighborhood of  $\alpha$ with respect to its trim value  $\alpha_0$  given at a specific trim airspeed  $V_T$ . Because aircraft deployed for pumping AWES are intended to fly over a wide rage of flight conditions, flight test maneuvers and parameter identification needs to be performed at multiple trim conditions. Fig. 4 shows the estimated pitch damping coefficient  $C_{m_q}$  related to AP2, as a function of  $\alpha$ with the corresponding value of  $C_{m_q}$  at  $V_T = 20$  m/s, the latter denoted in the aerospace field as trimmed coefficient.



Fig. 4. Estimated pitch damping coefficient  $C_{m_q}(\alpha)$  with corresponding trimmed coefficient for  $V_T = 20$  m/s

# 3. FORMULATION OF MULTIPLE EXPERIMENT PARAMETER ESTIMATION

Whenever parameter estimation is intended for identification of aircraft dynamics, multiple experiments are usually required to deal with the following issues:

- Reduce the effect of sensor biases as well as colored noise (atmospheric turbulence) on estimation results;
- Individual maneuvers might have good information content only for a subset of parameters, while multiple maneuvers taken together can provide a better information w.r.t the complete set of parameters;
- The flight test area and operating safety case restricts the flight paths that can be flown, limiting the available duration of any particular maneuver.

In this case, the estimated parameters can be retrieved via data fitting for each independent experiment and subsequently weighted w.r.t. to their inverse (estimated) covariance matrix  $\Sigma_{\theta^*}$  (Ljung, 1998). However, such method might lead to wrong results whenever computed  $\Sigma_{\theta^*}$  are not reliable. Furthermore, from (9,10,11) one can observe that angular acceleration measurements are required in order to retrieve estimates of *derivatives*. Usually, accelerations are not measured though, they can be retrieved by numerical differentiation methods from rates, which are noisy. Consequently, signal distortion may arise degrading the overall estimation performance (Morelli, 2006). According to what was mentioned above, multiple experiment MBPE algorithms might be beneficial for estimation of *aerodynamic derivatives*.

In this context, let us consider a mathematical model defined as a set of ODEs

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \theta, t)$$
(15a)

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t), \mathbf{u}(t), \theta, t)$$
(15b)

with differential states  $\mathbf{x} \in \mathbb{R}^{n_x}$ , output state  $\mathbf{y} \in \mathbb{R}$ control inputs  $\mathbf{u} \in \mathbb{R}^{n_u}$ , parameters  $\theta \in \mathbb{R}^{n_\theta}$ , and time t. A multiple experiments MBPE problem can be first stated using an optimal control problem (OCP) perspective in continuous time as follows

$$\underset{\theta}{\text{minimize}} \quad \sum_{i=1}^{N_e} \int_0^{T^i} \left( \frac{\hat{\mathbf{y}}^i(t) - \mathbf{h}\left(\mathbf{x}^i(t), \hat{\mathbf{u}}^i(t), \theta\right)}{\sigma_i^2} \right)^2 dt$$
(16a)

subject to  $\dot{\mathbf{x}}^{i}(t) = \mathbf{f}(\mathbf{x}^{i}(t), \hat{\mathbf{u}}^{i}(t), \theta, t)$  (16b)

$$t \in \left[0, T^i\right], \ i \in \mathbb{Z}_1^{N_e} \tag{16c}$$

with  $N_e$  number of experiments,  $\sigma_i$  noise variance,  $\hat{\mathbf{u}}^i(t)$ and  $\hat{\mathbf{y}}^i(t)$  respectively input, output measurements for  $i^{th}$  experiment running a length  $T^i$ . By means of *direct* methods (Diehl, 2014), (16) can be transformed into a finite dimensional nonlinear programming problem (NLP) which is then solved by numerical optimization methods. In this paper, we implemented a *direct multiple shooting* method (Bock and Plitt, 1984) since it is more stable with respect to the initial guess than a single shooting strategy. Let us define an equidistant grid over the experiment consisting in the collection of time points  $t_k$ , where  $t_{k+1} - t_k = \frac{T^i}{N_m^i} \coloneqq T_s, \forall i = 0, \ldots, N_e$  with  $N_m^i$  the number of measurements for  $i^{th}$ , assuming implicitly that the measurements are collected with a fixed sample time  $T_s$ . Additionally, we consider a piecewise constant control parametrization  $u(\tau) = u_k$  for  $\tau \in [t_k, t_{k+1})$ . A function  $\phi(.)$  over each shooting interval is given, which represents a numerical approximation for the solution  $x_{k+1}$  of the following initial value problem (IVP)

$$\dot{\mathbf{x}}(\tau) = \mathbf{f}(\mathbf{x}(\tau), \mathbf{u}_k, \theta, \tau), \ \tau \in [t_k, t_{k+1}]$$
(17)

Such function is evaluated numerically via integration methods, such as the Runge-Kutta of order 4 (RK4) as implemented in this paper. Thus, the OCP in (16) can be formulated into the following NLP

$$\underset{\theta, \mathbf{X}}{\text{minimize}} \qquad \sum_{i=1}^{N_e} \sum_{k=0}^{N_m} \left( \frac{\hat{\mathbf{y}}_k^i - \mathbf{h}\left(\mathbf{x}_k^i, \hat{\mathbf{u}}_k^i, \theta\right)}{\sigma_i^2} \right)^2 \qquad (18a)$$

subject to  $\mathbf{x}_{k+1}^i - \phi(\mathbf{x}_k^i, \hat{\mathbf{u}}_k^i, \theta) = 0$  (18b)

$$k \in \mathbb{Z}_0^{N_m - 1}, \ i \in \mathbb{Z}_1^{N_e} \tag{18c}$$

where  $\mathbf{X} \in \mathbb{R}^{n_{\mathbf{X}}}$  with  $n_{\mathbf{X}} = \sum_{i=1}^{N_e} n_x (N_m^i - 1)$  is sorted as

$$\mathbf{X} = [x_0^1, \dots, x_{N_m^1 - 1}^1, \dots, x_0^{N_e}, \dots, x_{N_m^e - 1}^{N_e}]^T$$
(19)

in order to ensure diagonal block structure on the NLP formulation. Notice that in (19) the number of measurements  $N_m$  are assumed different for each  $i^{th}$  experiment. The NLP initialization can be chosen e.g. from previous estimates of  $\theta$  while **X** can be initialized using the measurements  $\hat{\mathbf{y}}$  and/or estimates of the state  $\mathbf{x}$ . For further details refer to Bock et al. (2013).

# 4. DATA FITTING

4.1 Assessment of estimation performance via simulated flight tests

The *PowerPlane* is an autonomous aircraft, hence no action of the pilot occurs during the parameter identification flight test unless system failures are detected. During the design of maneuvers, reliable simulators play an important role both for the assessment of estimation performance and especially to prevent violation of flight envelope due to aggressive maneuvers.

For the sake of simplicity, let us consider the longitudinal dynamics (10), assuming  $J_{xz} = 0$  since  $J_{xz}$  is only the 1.8% of the smallest moment of inertia related to AP2, which is  $J_x$  (see Table A.1 in the Appendix). Experimental data are generated from a full 6 DOF AP2 simulator by injecting feasible time based excitation signals type 3-2-1-1 with different amplitudes and pulse width. For a more detailed description of input signals and their rationale behind, the reader is referred to Mulder et al. (1994). During the excitation signals, the aircraft is in gliding mode  $(F_p = M_p = 0)$  while roll and yaw rate are kept constantly trimmed (p = r = 0) by feedback control. In the simulation environment all gusts and turbulence was turned off. The logged simulated outputs are then corrupted with white Gaussian noise with standard deviation  $\sigma$  shown in the appendix Table A.2 and in agreement with the available sensors. The controllable input i.e. the elevator deflection  $\delta_e$  has no discernible noise, though quantization errors are present and equal to 0.25 deg. According to the assumption taken into account, the model used for data fitting is

$$\dot{q} = \frac{\frac{1}{2}\rho V^2 \cdot \left[C_{m_{\alpha}}\alpha + C_{m_q}\frac{\bar{c}q}{2V} + C_{m_{\delta_e}}\delta_e + C_{m_0}\right]}{J_u} \quad (20)$$

with

$$\hat{\mathbf{y}}(t) = \mathbf{x}(t) = q$$

$$\hat{\mathbf{u}}(t) = [V, \alpha, \delta_e]^T$$

$$\theta = [C_{m_0}, C_{m_\alpha}, C_{m_{\delta_e}}, C_{m_q}]^T$$
(21)

The multiple experiment parameter estimation problem (18) is constructed in Matlab as a CasADi (Andersson, 2013) computational graph and solved with IPOPT (Wächter and Biegler, 2006). CasADi discovers the structure shown in Fig. 5 and computes the full sparse Jacobian and Hessian with a minimal of algorithmic differentiation sweeps. CasADi's for-loop equivalents are used to efficiently build up the large number of dynamic constraints (18b). Furthermore, since this application requires a large number of control intervals, the CasADi map functionality was used to achieve a memory-lean computational graph. For this specific case, solution  $w_{opt}$  was found after 6 iterations with 3912 samples and  $T_s = 10$ . Both simulation experiments as well as data fitting are shown in Fig. 6 while the deviation in percentage of estimates  $\theta^*$  with respect to the true values  $\theta^0$  are collected in Table 1. One can observe that all estimates are within the satisfactory engineering accuracy.

# Table 1. Estimation results: simulation case (values in percentage)

#### 4.2 Confidence ellipsoids and turbulence effect

One way to assess the quality of the estimation results  $\theta^*$  is by means of 1- $\sigma$  confidence ellipsoids and in short they help to assess the probability that the true value  $\theta^0$  is contained in the set  $\varepsilon_1(\theta^*)$  defined by

$$\varepsilon_1(\theta^*) := \{ \theta \in \mathbb{R}^d \, | \, \|\theta - \theta^*\|_{\Sigma_{\theta^*}}^2 \leqslant 1 \}$$
(22)

with d the dimension of the parameter space. The 1- $\sigma$  confidence ellipsoids can be computed from an estimate of the covariance matrix  $\Sigma_{\theta^*}$ . The uncertainties of each parameter is retrieved from the diagonal entries of  $\Sigma_{\theta^*}$  while the correlation over the parameters is characterized by the non-diagonal entries. For more details the reader is



Fig. 5. Jacobian and Hessian Sparsity of the NLP



Fig. 6. multiple experiment and data fitting: simulation case



Fig. 7. 1- $\sigma$  confidence ellipsoid for the pair  $(C_{m_{\delta_e}}, C_{m_{\alpha}}^*)$ . The case with turbulence is performed using the same boundary condition described in section 4.1 but with turbulence switch on. Turbulence are generated by *Von Karman* turbulence model.

referred to (Diehl, 2015) for the single experiment case and (Bock et al., 2013; Morelli, 2006) for the multiple experiment case. At any rate, whenever significant process noise is present, 1- $\sigma$  confidence ellipsoids computed by the estimation algorithm presented in section 3 might not be reliable as shown in Fig. 7, where the 1- $\sigma$  confidence ellipsoid for the pair of estimates ( $C_{m_{\delta_e}}, C_{m_{\alpha}}$ ) is provided with and without the presence of turbulence.

# 4.3 Data fitting via real flight experiments

After the assessment of estimation accuracy via a priori models, a set of experimental data was retrieved from flight tests using AP2 with trim airspeed  $V_T = 20$  m/s. During the flight test, six signal input 3-2-1-1 type were performed along the longitudinal dynamics. Unfortunately, wind speed was rather consistent with an average of  $\approx 10$  m/s, which is not optimal for system identification purposes with an aircraft that flies at 20 m/s. Among the six maneuvers, one was discarded due to dominant turbulences with respect to excitation signal while four experiments were used as *estimation data* and one as *validation data*. The two maneuvers designed and shown in Section 4.1 were computed and collected in the estimation data set.

In this study, the full pitch dynamics was taken into account, where the model is assembled using eq. (10),(12e)implementing the same criteria described in Section 4.1 with the difference that  $J_{xz} \neq 0$  and  $\hat{\boldsymbol{u}} = [p, r, V, \alpha, \delta_e, \bar{q}]^T \in$  $\mathbb{R}^6$ . Although the inclusion of  $\bar{q}$  in  $\hat{\boldsymbol{u}}$  might appear redundant (since  $\bar{q} = \frac{1}{2}\rho V^2$ ,  $\rho \approx 1.23$ ), flight computer control (FCC) computes dynamic pressure measurements  $\hat{q}$  considering an estimate of the air-density  $\hat{\rho}$  which is function of several parameters e.g. temperature and altitude, providing in this way additional accuracy. Furthermore, measurements were suitably low-pass filtered using zero-lag filtering since we are interested in the rigid-body modes only and inertia values are assumed to be known and equal as in Table A.1 in the appendix. The control surface inputs are measured via feedback sensors on the aircraft, which allows the estimation to proceed without requiring knowledge of the actuator dynamics. However, a one frame transport delay of the measurements was used.

Fig. 8 shows one system identification flight test, where the data fitting is computed right after that the throttle percentage  $\delta_t$  is set to zero. The excitation signal was injected during the open-loop phase while aileron and rudder stabilize respectively roll and yaw dynamics (see fig. 9). Note that the last data in the open-loop phase were omitted from the data fitting because it was found to skew the result, apparently due to the contribution of turbulence to the dynamic response. Finally, data fitting of the whole estimation data set are shown in terms of residual distribution  $\epsilon$  which is defined as follow (fig. 10):

$$\epsilon_k = \hat{y}_k - \mathbf{h} \left( \mathbf{x}_k, \hat{\mathbf{u}}_k, \theta^* \right), k = 1, \dots, N_m$$
  
$$\theta^* = \left[ Cm_\alpha^*, C_{m_{\delta_a}}^*, C_{m_a}^*, C_{m_0}^* \right]^T$$
(23)

Practically speaking, the residual is the part of the data that the model is not able to reproduce; the aim is to achieve a residual resembling a white noise signal. At any rate, it is well-known that the residuals will not be white noise if the real system has significant process noise (turbulence).

# 4.4 Assessment of model and estimation results

There are severals ways to assess the goodness of model structure. One straightforward way is to perform a forward simulation given by the selected model combined with the estimates  $\theta^*$  along the validation data set. Fig. 11 shows the comparison between the forward simulation with respect to pitch rate response and respective residual distribution. Another indicator for determining the *goodness* 



Fig. 8. Data fitting along longitudinal dynamics: real flight experiment

of fit is given by the so called R-squared  $(R^2)$  value which is represented by the following expression

$$R^{2} = 1 - \frac{\sum_{k=1}^{N_{v}} (\hat{y}_{k} - \mathbf{h} (\mathbf{x}_{k}, \hat{\mathbf{u}}_{k}, \theta^{*}))^{2}}{\sum_{k=1}^{k=N_{v}} \hat{y}_{k}^{2}}$$
(24)

with  $N_v$  number of measurements related to validation data set. The  $R^2$  value is always between zero and one, often expressed in percentage and it is independent from the problem data size. A value of one means perfect fit while a zero value means that model is not able to explain any of the data. For this specific model structure,  $R^2$  was around 93%.

As far as it regards the quality of estimation performance, the confidence ellipsoids show reasonable uncertainties on the estimates (see Fig 12). However, due to the presence of turbulence during the flight tests, computed covariance may not have been very reliable as mentioned in section 4.2. In particular, it turned out that  $C_{m_q}^*$  differs widely from previous experiments as well as from numerical methods, which will be addressed with future studies, validation and design of proper excitation signals.



Fig. 9. Stabilization of lateral dynamics by  $\delta_a$  and  $\delta_r$  during excitation signal along longitudinal dynamics



Fig. 10. Residual distribution of four remaining independent experiments with corresponding mean value  $\mu$ and standard deviation  $\sigma$ : data fitting on pitch rate q

# 5. CONCLUSION

In this paper, we have shown theoretical and practical aspects related to identification of *aerodynamic derivatives* by means of real flight tests related to *pumping* AWES. An MBPE algorithm has been implemented for handling multiple experiments using a large scale optimization algorithm. The reliability of both estimates and mathematical model have been assessed by tool such  $R^2$  value and confidence ellipsoids. The results highlight the importance of conducting flight tests in a low disturbance environment to minimize the effects of process noise.



Fig. 11. Model structure assessment via validation data set



Fig. 12. 1- $\sigma$  confidence ellipsoids of pairs formed among  $\theta^* = [Cm^*_{\alpha}, C^*_{m_{\delta_e}}, C^*_{m_q}, C^*_{m_0}]$ 

# ACKNOWLEDGEMENTS

The authors would like to thank Ampyx Power, and in particular the flight operations team for conducting system identification flight tests.

# Appendix A. TABLES

Table A.1. parameters PowerPlane  $2^{nd}$  generation

Name	Symbol	Value
mass	m	36.8kg
inertia	$J_x, J_y, J_z, J_{xz}$	$25, 32, 56, 0.47 \text{ kg} \cdot \text{m}^2$
reference wing area	S	$3 \text{ m}^2$
reference wing span	b	$5.5 \mathrm{m}$
reference chord	$\overline{c}$	$0.55 \mathrm{m}$

Table A.2. Available sensors with corresponding noise standard deviation  $\sigma$ 

Sensor	Variable	$\sigma$
IMU	q	0.1  deg/s
Pitot tube	V	$3.6 \mathrm{~m/s}$
Air Probe	$\alpha$	$0.5 \deg$

# REFERENCES

- Anderson Jr, J.D. (2010). Fundamentals of aerodynamics. Tata McGraw-Hill Education.
- Andersson, J. (2013). A General-Purpose Software Framework for Dynamic Optimization. PhD thesis, Arenberg Doctoral School, KU Leuven, Department of Electrical Engineering (ESAT/SCD) and Optimization in Engineering Center, Kasteelpark Arenberg 10, 3001-Heverlee, Belgium.
- AP (2016). Ampyx power: Airborne wind energy. URL https://www.ampyxpower.com/.
- Bock, H.G., Carraro, T., Jäger, W., Körkel, S., Rannacher, R., and Schlöder, J. (2013). Model Based Parameter Estimation: Theory and Applications, volume 4. Springer Science & Business Media.
- Bock, H.G. and Plitt, K.J. (1984). A multiple shooting algorithm for direct solution of optimal control problems. Proceedings of the IFAC world congress.
- Diehl, M. (2013). Airborne wind energy: Basic concepts and physical foundations. In *Airborne Wind Energy*, 3–22. Springer.
- Diehl, M. (2014). Optimal Control and Estimation (lecture notes). University of Freiburg.
- Diehl, M. (2015). Modelling and System Identification (lecture notes). University of Freiburg.
- Goman, M. and Khrabrov, A. (1994). State-space representation of aerodynamic characteristics of an aircraft at high angles of attack. *Journal of Aircraft*, 31(5), 1109– 1115.
- Gros, S. and Diehl, M. (2013). Modeling of airborne wind energy systems in natural coordinates. In *Airborne wind* energy, 181–203. Springer.
- Horn, G., Gros, S., and Diehl, M. (2013). Numerical trajectory optimization for airborne wind energy systems described by high fidelity aircraft models. In *Airborne* wind energy, 205–218. Springer.
- Licitra, G., Sieberling, S., Engelen, S., Williams, P., Ruiterkamp, R., and Diehl, M. (2016). Optimal control for

minimizing power consumption during holding patterns for airborne wind energy pumping system. Proceedings of the European Control Conference.

- Ljung, L. (1998). System identification: Theory for the user. Prentice Hall PTR.
- Loyd, M.L. (1980). Crosswind kite power (for large-scale wind power production). Journal of energy, 4(3), 106– 111.
- Morelli, E.A. (2006). Practical aspects of the equationerror method for aircraft parameter estimation. *AIAA* paper, 6144, 2006.
- Mulder, J., Sridhar, J., and Breeman, J. (1994). Identification of dynamic system-application to aircraft nonlinear analysis and manoeuvre design. Technical report, Technical Report AG 300, AGARD.
- Mulder, J., Van Staveren, W., and van der Vaart, J. (2000). Flight dynamics (lecture notes): ae3-302. TU Delft.
- Versteeg, H.K. and Malalasekera, W. (2007). An introduction to computational fluid dynamics: the finite volume method. Pearson Education.
- Wächter, A. and Biegler, L.T. (2006). On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Mathematical* programming, 106(1), 25–57.
- Williams, P., Lansdorp, B., and Ockels, W. (2007). Modeling and control of a kite on a variable length flexible inelastic tether. In *AIAA Guidance, navigation and control conference.*
- Williams, P., Lansdorp, B., Ruiterkamp, R., and Ockels, W. (2008). Modeling, simulation, and testing of surf kites for power generation. American Institute of Aeronautics and Astronautics.
- Zanon, M., Gros, S., and Diehl, M. (2013). Model predictive control of rigid-airfoil airborne wind energy systems. In *Airborne Wind Energy*, 219–233. Springer.